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Development of microstructure in FCC metals during cold work

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The evolution in microstructure and local crystallography is described, with emphasis on the behaviour of medium to high stacking fault energy FCC metals deformed at low temperature. This evolution is analysed within a common framework of grain subdivision on different size-scales. The largest scale is the macroscopic subdivision of crystals or grains, which will be presented using the behaviour of single crystals as an example. Subdivision on smaller scales is demonstrated for polycrystals, with emphasis on the effects of strain and grain orientation on the evolution in microstructure and local crystallography. This leads to an introduction of structural parameters and their analysis using a scaling hypothesis. A key finding is the correlation between structure and crystallographic orientation, which allows a slip pattern description by standard crystal plasticity models, leading to an analysis of the relationship between slip pattern and microstructure. These findings are briefly related to the macroscopic properties of deformed metals.

Keywords: grain subdivision; slip pattern analysis; scaling hypothesis; orientation microstructure relationship; low-energy structure; flow stress anisotropy

1. Introduction

The study of microstructural evolution during cold work has a long tradition, and steps forward have frequently been associated with the application of new techniques. This has also been the case in the last decade, where automated microscopical techniques and fast computational methods have been introduced. This development now allows both observation and modelling to be carried out from the macroscopic to the microscopic level. Thereby, the use of structural information to link processing conditions and the macroscopic properties of metals and alloys is within reach. In this paper, the evolution in microstructure and local crystallography is discussed with emphasis on the behaviour of medium to high stacking fault energy (SFE) single phase FCC metals, deformed in monotonic deformation modes at low temperature where diffusional processes (climb for example) can be neglected.

2. Plastic deformation

During plastic deformation, most of the mechanical energy is converted into heat and only a fraction is stored in the metal, mostly in the form of dislocations. These dislocations are not randomly distributed. They accumulate in dislocation boundaries, which separate regions with a relatively low dislocation density. A distinctive example is the formation of dislocation cells (Swann 1963).

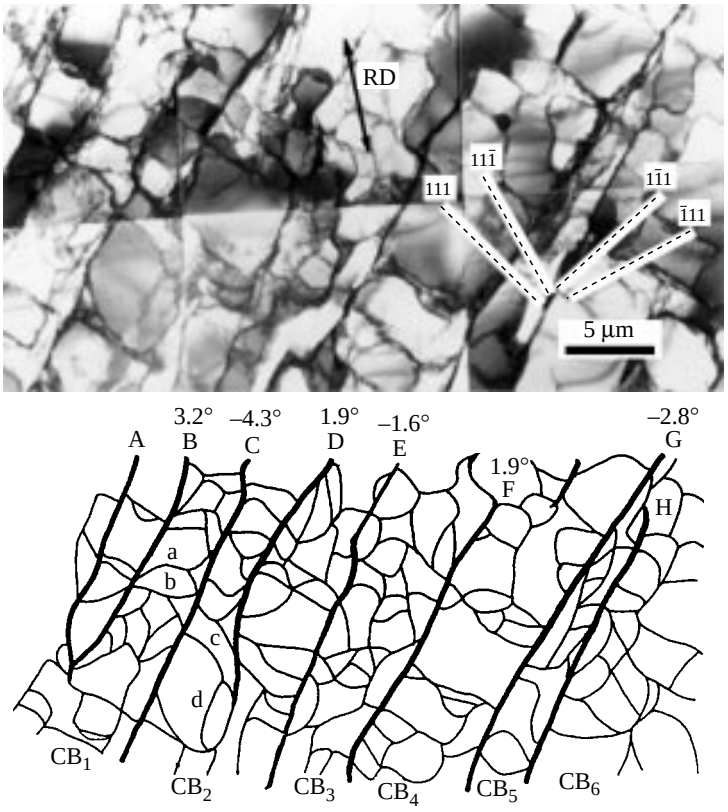

Figure 1. (*a*) A TEM image and a sketch of a microstructure in a grain of a 10% cold rolled specimen of pure aluminium (99.996%) in longitudinal plane view. One set of extended non-crystallographic dislocation boundaries is marked A, B, C, etc., and their misorientations are shown. These boundaries form cell blocks marked CB_1, CB_2, etc., which are subdivided by cell boundaries marked a, b, c, etc. The rolling direction is marked RD and the dashed lines are traces of {111} planes (Liu & Hansen 1995).

The accumulation of dislocations in dislocation boundaries renders these boundaries the key feature in the characterization of deformation microstructures. Dislocation cells and cell boundaries have been studied for many years, but recent analyses of deformation microstructures in single crystals and polycrystals have shown that cell boundaries coexist with dislocation boundaries, which have a significantly different morphology (see figure 1). In contrast to the short cell boundaries with an almost random orientation in the structure, these other boundaries are long nearly planar boundaries. These extended boundaries have varying characteristics, but a common feature is that they delineate regions that are further subdivided by ordinary cell boundaries (Kuhlmann-Wilsdorf 1989; Hansen 1990; Bay *et al*. 1992; Hughes 1995).

The two types of boundaries may be formed by different mechanisms. The cell boundaries may form by mutual trapping of glide dislocations and, as such, have been termed 'incidental dislocation boundaries' (IDBs) (Kuhlmann-Wilsdorf & Hansen 1991). Extended boundaries may have their origin in a different range of active slip systems in neighbouring regions called cell blocks (see figure 1). Each cell block has been assumed (Bay *et al*. 1992) to deform by four or less active slip systems,

i.e. falling short of the five required for homologous deformation according to the Taylor model (Taylor 1938). However, a group of cell blocks may collectively fulfil the Taylor criterion. This slip pattern will cause a strain in the cell block that is different from the macroscopic strain. Strain differences are accommodated by the formation of cell-block boundaries which, therefore, have been termed 'geometrically necessary boundaries' (GNBs) (Kuhlmann-Wilsdorf & Hansen 1991)†. It has been suggested that the number of simultaneously acting slip systems is a result of two competing effects (Bay *et al*. 1992). The flow stress is lowered with a reduction in slip systems because the number of intersecting dislocations and, thus, the number of jogs, decreases. On the other hand, stress screening will become more effective as the number of different Burgers vectors increases (Kuhlmann-Wilsdorf 1989).

The slip pattern discussed above is not the only one that can cause grain subdivision and formation of GNBs. The slip pattern may also follow the Taylor model, i.e. the strain in the different regions in a grain is the same as the macroscopic strain, but GNBs can form if different sets of slip systems operate in neighbouring regions or if the total shear amplitude is partitioned differently among a common set of systems in neighbouring regions (Wert 1998). Finally, GNBs may form due to a non-uniform strain in the interior of the grains (see, for example, Becker & Panchanadeesvaran 1995), or near grain boundaries and triple junctions (Barlow *et al*. 1985; Randle *et al*. 1996; Zisman & Rybin 1998).

A grain subdivision on different length-scales is most characteristic in wavy (distributed) glide materials, such as FCC metals with medium to high SFE, e.g. Cu, Ni and Al (Malin & Hatherly 1979; Bay *et al*. 1992; Hughes & Hansen 1993; Huang 1998), and BCC metals, e.g. iron (Jago & Hansen 1986). In these metals, a wavy slip band structure indicates a three-dimensional mobility of dislocations by cross-slip. However, grain subdivision (although more difficult to observe) also takes place in planar glide materials such as low-SFE metals, e.g. austenitic stainless steel (D. A. Hughes 1998, personal communication) and some alloys, e.g. Al–Mg (Hughes 1993). A typical microstructure for a wavy glide material is seen in figure 1, where the structure is subdivided into cell blocks and cells. Figure 2 shows grain subdivision in a planar glide material (warm rolled Al–5%Mg), where the extended dislocation walls now form blocks, which, instead of cells, contain a more uniform distribution of dislocations called a Taylor lattice. The microstructures shown in figures 1 and 2 are typical deformation structures. However, a change in process parameters can affect the slip pattern and the deformation microstructure. For example, in wavy glide metals, the tendency for planar glide increases with decreasing temperature (and

† The classification of dislocation boundaries into IDBs and GNBs is often compared to the grouping of stored dislocations into statistically stored dislocations and geometrically necessary dislocations (Cottrell 1963; Ashby 1970). The former type remains after deformation due to a random accumulation process, whereas the latter type is necessary to accommodate a non-uniform strain. A similar distinction has been made by Weertman (1998) who, like Cottrell (1963), divides the dislocations in a deformation process into redundant dislocations (of opposite sign) and non-redundant dislocations, which are collected due to a non-uniform plastic deformation. This means that IDBs, in principle, are composed of statistically stored or redundant dislocations, and that GNBs are composed of geometrically necessary or non-redundant dislocations, if their cause is a non-uniform strain in the grain. However, GNBs can also have other causes, as is discussed in this section. At the medium to large strains considered in this paper, both IDBs and GNBs are rotational dislocation boundaries being characterized by an axis and angle of misorientation. Some of the dislocations in these boundaries produce the misorientations. In Wert *et al*. (1995), these dislocations have been termed geometrically necessary boundary dislocations. These dislocations are supplemented by redundant boundary dislocations.

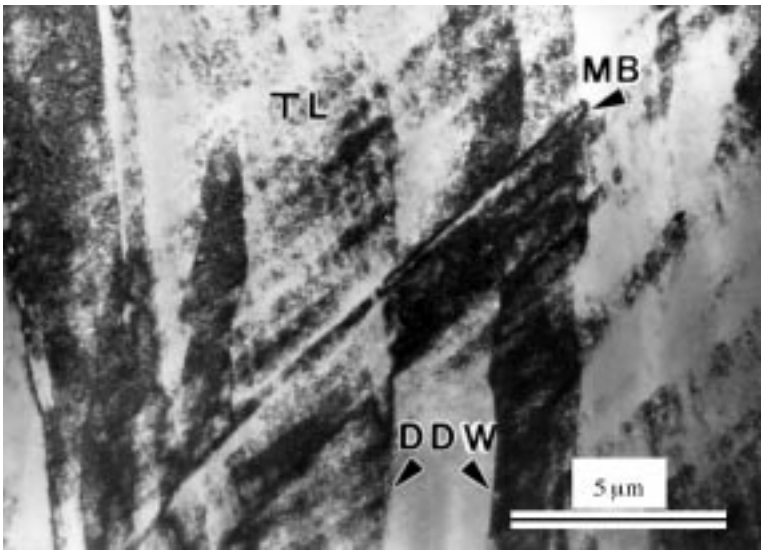

Figure 2. Dislocation microstructure in Al–5%Mg warm rolled at 350 °C to a 30% reduction in longitudinal plane view. Extended boundaries are seen that form a Taylor lattice (TL) block structure. The extended boundaries are marked DDW and MB (see text), respectively (Hughes & Godfrey 1998).

increasing strain rate), whereas a planar glide metal after hot deformation shows microstructures typical of wavy glide. However, under all conditions, the subdivision into a block structure is characteristic (Liu *et al*. 1998; Theyssier *et al*. 1995; Duly *et al*. 1996; Hughes & Godfrey 1998; Juul Jensen *et al*. 1998). In the following, we shall concentrate on the subdivision in medium to high stacking fault FCC metals deformed at room temperature, i.e. metals that form a cell-block structure.

3. Grain subdivision

The subdivision of crystals and grains during deformation takes place on a macroscopic (or grain) scale into deformation bands, and on smaller scales into cell blocks and cells.

(*a*) *Macroscopic subdivision*

The macroscopic subdivision of crystals and grains has been investigated since the discovery by Barrett & Levenson (1940) of deformation bands in cold compressed single crystals and polycrystals of aluminium. Many different techniques have been applied, and the subdivided structure is generally described as being composed of well-defined wide deformation bands (or matrix bands) separated by more narrow regions, termed transition bands. During deformation, the matrix bands rotate in different directions and orientation differences between neighbouring bands are accommodated in transition bands characterized by a relatively high cumulative misorientation across them (Hu 1963; Dillamore *et al*. 1972). For reviews in this area, see, for example, Cottrell (1953), Chin (1973), Gil Sevillano *et al*. (1981), Doherty (1980), Humphreys & Hatherly (1995), Liu & Hansen (1998) and Wert (1998). For a recent

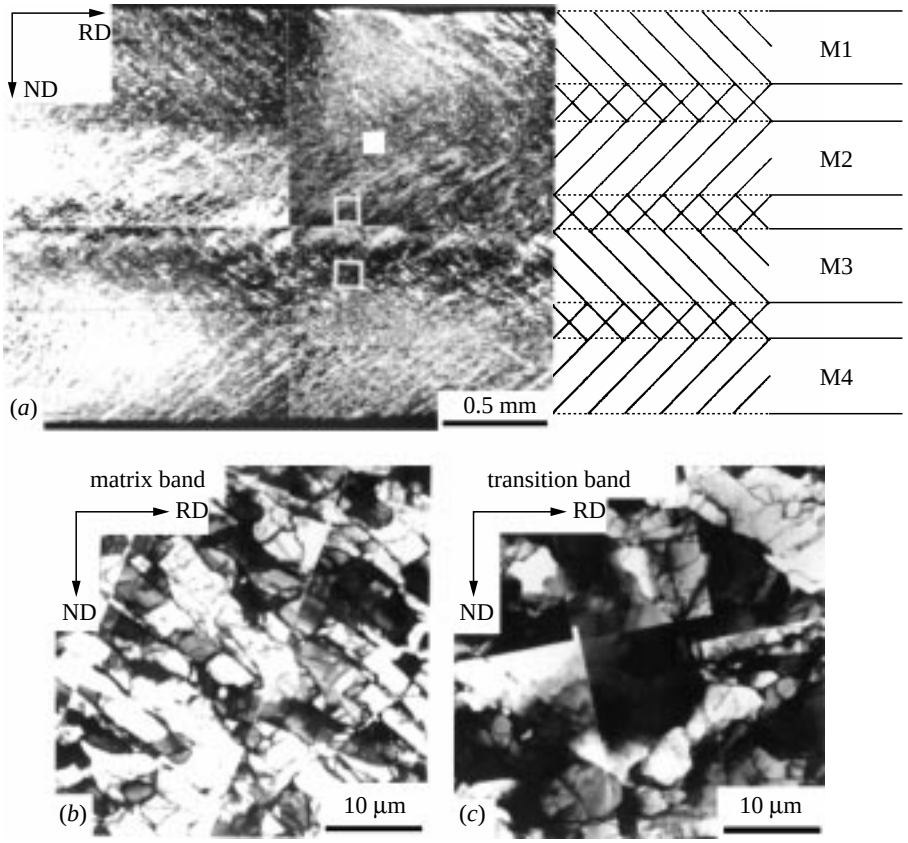

Figure 3. Subdivision of a cube-oriented aluminium crystal cold rolled 30%. (*a*) SEM channelling contrast image showing the subdivision of the crystal into four matrix bands marked M1, M2, etc., and three transition bands. (*b*) and (*c*) TEM micrographs revealing the microstructure in a matrix and a transition band are marked by squares in (*a*). The rolling direction and the normal direction are marked RD and ND, respectively (Liu & Hansen 1998).

analysis of deformation-band formation, especially as a function of grain size, see Lee & Duggan (1993).

The many experimental conditions, and an inconsistent nomenclature, make it difficult to synthesize the many observations of the macroscopic break-up of crystals and grains. However, a number of single-crystal experiments have been analysed in detail and have shown a consistent picture of the macroscopic break-up, which will be illustrated by some examples.

The behaviour of an aluminium crystal of cube orientation (001)[100] cold rolled to different reductions has been analysed by Wert *et al.* (1997) and Liu & Hansen (1998). Both scanning electron microscopy and transmission electron microscopy have been applied, allowing observations over several scales of magnification. It has been observed that by opposite rotations around the transverse direction (TD), the crystal has subdivided into four matrix bands, which are parallel to the rolling plane. Between the four bands there are three transition bands, in which the orientation changes continuously from that of a matrix band to that of an adjoining band (see

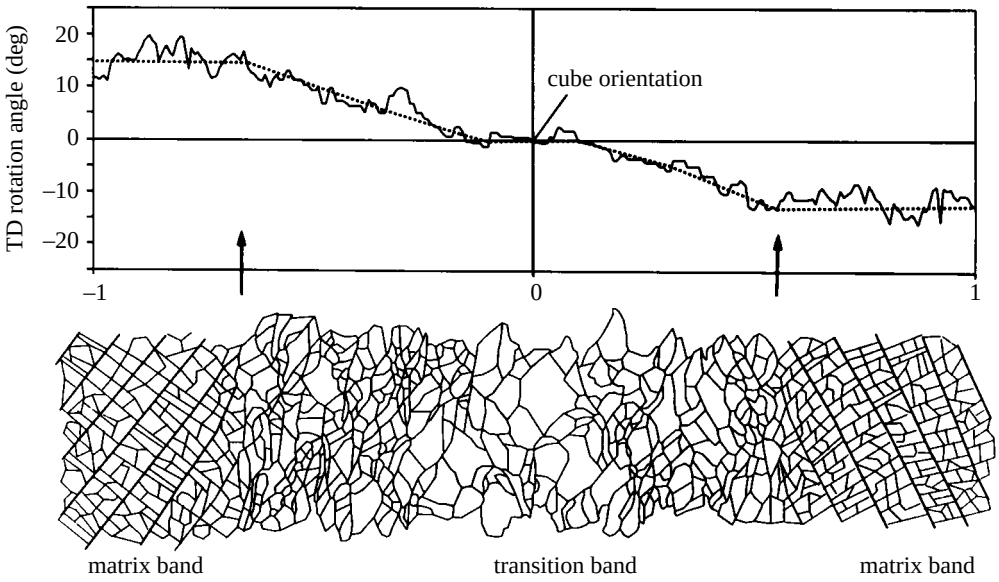

Figure 4. A schematic display of the dislocation structure morphology in figure 3 together with the rotation angle around TD. The extension of a transition band is marked by two arrows and it can be seen that the original cube orientation is maintained in the middle of the transition band. The measurements of the rotation angle have been made by Kikuchi pattern analysis in a TEM over a distance of *ca.* 200 μm in the ND direction in the ND–RD plane.

figure 3). This figure also shows that a cell structure characterizes the transition bands and that a cell-block structure is formed in the matrix bands. This difference is illustrated schematically in figure 4, which also shows the rotation angle around the TD. For homogeneous rolling conditions, this break-up has been analysed with respect to the slip pattern where a Schmid analysis shows that two pairs of collinear slip systems are equally favoured. If the shear-strain amplitude is the same on all four systems, the crystal is stable under plane-strain conditions. However, an unequal division of the total shear amplitude between the two pairs produces rotation of the crystal. The apportioning scheme, which produces a TD rotation, creates a non-zero shear strain ε_{13}, where the RD, TD and ND are numbered 1, 2 and 3, respectively. The relation between a shear amplitude difference and a shear strain ε_{13} leads to an analysis of the location dependency of this shear strain, a problem that has been analysed by Lee & Duggan (1991). In their modelling of inhomogeneous rolling textures, they suggest that both the rolling geometry and the friction effect can introduce a location-dependent shear strain, and that the combined effect of geometry and friction can cause the sign of this shear strain (ε_{13}) to alternate through a rolled specimen along the ND. A correlation may therefore exist between ε_{13} and the rotation angle around TD, and it is demonstrated that the variations in ε_{13} through the specimen thickness qualitatively agree with the change in rotation angle around TD, and with the break-up of the crystal into four matrix bands and three transition bands (Liu & Hansen 1998). According to this analysis, the slip pattern differs between the bands, leading to different deformation microstructure, which is illustrated in figures 3 and 4 showing the dislocation structure morphology in a transition band and in a neighbouring matrix band. In recent (unpublished)

work, this analytic examination has been extended to cold rolled crystals of the Goss orientation and the rotated cube orientation. In both cases, a location-dependent shear strain is found to be a decisive parameter.

The good agreement between a location-dependent shear strain and a macroscopic break-up of the crystals has also been found by Wonsiewicz & Chin (1970), who analysed a Cu crystal of $(111)[\bar{1}\bar{1}2]$ orientation deformed in plane-strain compression. In this study, it was suggested that friction effects produce a location-dependent shear strain, which favours one or the other of two initially equally favoured pairs of slip systems. As a result, the crystal breaks up into fragments that rotate to either $(112)[\bar{1}\bar{1}1]$ or $(110)[001]$.

The macroscopic break-up of single crystals into regions that extend over several hundreds of micrometres reflects rotation changes on a macroscopic scale. Such changes can be caused by the imposed deformation conditions (e.g. geometry and friction), which may also cause macroscopic subdivision in polycrystals with a large grain size. However, as the grain size becomes smaller, conditions for macroscopic rotation changes will be difficult to obtain. For such smaller grain sizes it is possible that grain interaction may start to play a role. Such effects of local neighbourhood have been investigated in a number of finite-element simulations; see, for example, Becker *et al*. (1991), Becker & Panchanadeesvaran (1995), Beaudoin *et al*. (1996), Mika & Dawson (1998, 1999) and Dawson & Beaudoin (1998). In the following, some results will be discussed.

In general, finite-element simulations of subdivision of grains during deformation require a finely discretized mesh in order to resolve the deformation gradients within the grains. This effect of mesh refinement has been studied by Mika & Dawson (1998) who found, in plane-strain compression, that an increase in the number of elements from 48 (1099 grains) to 576 (172 grains) within each grain enhanced the calculated difference in crystal deformation between the centre of the grains and the region near the grain boundary. A better correlation is thereby obtained between simulation and experiments (see § 3 *b*). For the same polycrystals, the grain subdivision has been studied by a characterization of the boundaries separating regions within grains developing different lattice orientations (Mika & Dawson 1999). It has been found that the average angle across the interior boundaries increases with strain (2/3 power), which agrees with experimental results for GNBs (Hughes 1995). However, the interior boundaries in the simulation (Mika & Dawson 1999) are oriented close to the compression plane and have a spacing that is a large fraction of the grain size. This simulation, therefore, contradicts experiments where, for instance, in rolling, the majority of the GNBs have their planes *ca.* 40° to the rolling plane and a spacing of the order of micrometres (Bay *et al*. 1989; Hughes & Hansen 1993; Christoffersen & Leffers 1998).

An important part of finite-element simulation is to validate the results by comparison with experiments. In such research, it may be beneficial to study systems where the imposed strain conditions and the resulting slip pattern can be analysed. Single crystals and bicrystals are suitable for such studies. Detailed studies of polycrystals to investigate the effect of grain interaction are also of interest, as, in a few cases, characteristic matrix bands/transition bands have been observed to subdivide grains of a size of the order of 100 μm (Huang *et al*. 1998; and see figure 5). Part of such investigations is the effect of grain orientation on the deformation microstructure, which is discussed further in the next sections.

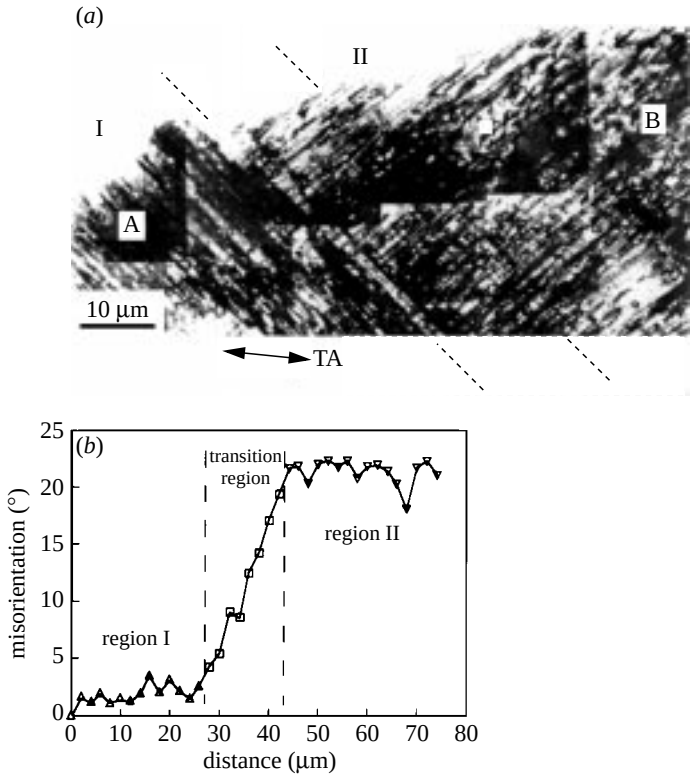

Figure 5. (*a*) A TEM image showing the deformation microstructure of a grain in a copper polycrystal (grain size 90 µm) deformed to a strain of 23% in tension. On a large scale, the grain split into two regions, I and II, connected by a narrow transition region bounded by two dashed lines. Inside each region, the subdivision takes place on smaller scales into cell blocks and cells with one set of extended boundaries (DDWs/MBs) being dominant. Note that the DDWs/MBs in regions I and II are nearly perpendicular to each other, and that they intersect in the transition region. (*b*) Orientation scanning was made in the TEM along a line from point A to point B with an interval of 2 µm. The angles have been determined between the first point and the rest of the points and it is seen that the grain has subdivided into two matrix bands and a transition band (Huang *et al*. 1998).

(*b*) *Microscopic subdivision and local crystallography*

Many microstructural studies have shown that the cell blocks are the typical building blocks that form the deformation microstructure (see, for example, Bay *et al*. 1992). In figures 1 and 6, a typical microstructural evolution in aluminium is illustrated showing the change in the cell-block structure with increasing strain. Note that cell-block (dense dislocation walls (DDWs) and microbands (MBs)) boundaries at low and medium strains have boundary planes that follow certain planes; either a slip plane or a plane that is not parallel to a slip plane. In both cases, the boundaries are macroscopically oriented with respect to the specimen axes. These figures also show that with increasing strain, the cell-block size and the cell size decrease; the former at a higher rate than the latter. As a result, the number of cells per cell block decreases (Liu & Hansen 1995). At large strain, the structural evolution

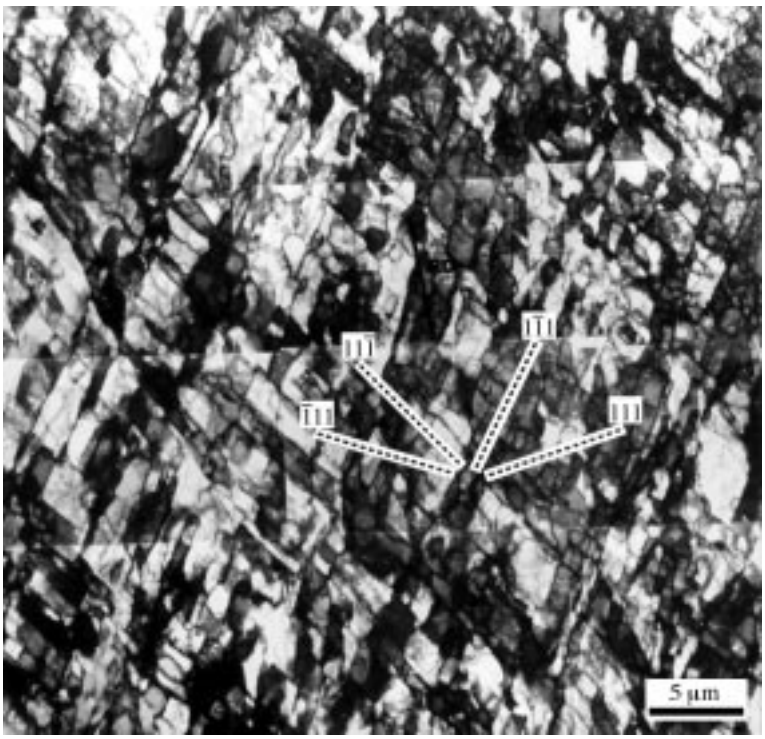

Figure 6. A TEM image of a microstructure in a grain of pure aluminium cold rolled 50% in longitudinal plane view. The microstructure is subdivided into a cell-block structure as shown in figure 1, but on a finer scale. Two sets of DDWs/MBs, which are on $(11\bar{1})$ and $(1\bar{1}1)$, have developed symmetrically with respect to RD (Liu *et al*. 1998).

brings the cell-block boundaries into a macroscopic orientation almost parallel to the rolling plane (see figure 7). Lamellar boundaries are formed bounding cell blocks that, on average, consist of only one row of cells (Malin & Hatherly 1979; Hansen & Hughes 1995; Hughes & Hansen 1997). The refinement of the microstructure is followed by an increase in the average angle across cell-block boundaries and cell boundaries. Detailed analyses show that the deformation-induced boundaries display a large angular spread and can reach average values of 5–10° at intermediate strains, and that the angular distributions at large strain contain a significant fraction of deformation-induced high-angle boundaries (Gil Sevillano *et al*. 1981; Liu *et al*. 1998; Juul Jensen 1995*a*, *b*; Hughes & Hansen 1997; Oscarsson *et al*. 1994; also see figure 8). This histogram also shows that many low-angle boundaries are present, i.e. new cell boundaries are being continuously formed during the deformation.

The microstructural evolution has been analysed (Hansen & Hughes 1995) in accordance with the hypothesis that the deformation structures are low-energy dislocation structures (LEDSs); see § 5. Recently, this principle has been used in an analysis of the behaviour of GNBs in rolled FCC metals (Pantleon & Hansen 1998). These boundaries are observed to have an angle of *ca.* 40° with the rolling direction in the longitudinal plane, and, with increasing strain, this angle decreases only slightly, in contrast to ordinary grain boundaries that rotate much faster towards the rolling plane (Bay *et al*. 1989; Hughes & Hansen 1993; Christoffersen & Leffers 1998). This

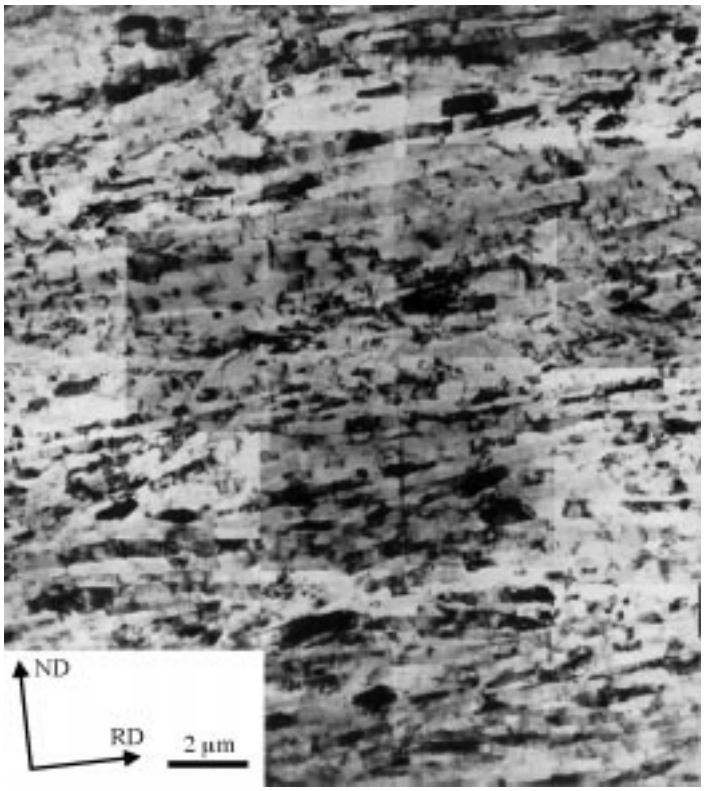

Figure 7. A TEM image of a microstructure in a grain of commercial-purity aluminium cold rolled to a strain of 5 (99.3% in longitudinal plane view). Most of the lamellar boundaries are almost parallel to the trace of the rolling plane (courtesy of Q. Liu).

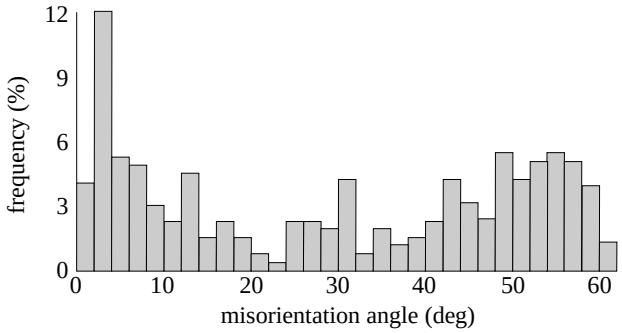

Figure 8. The frequency of misorientation angles across boundaries subdividing the structure in figure 7. This diagram is based on 265 measurements giving on average an angle of 29.2°. About two-thirds of the boundaries can be characterized as high-angle boundaries (greater than 15°) (courtesy of Q. Liu).

behaviour has been explained, by considering the operating slip systems and their effect on the plastic deformation, in a model that accounts for the persistence of the dislocation boundaries as being due to the tendency of the boundaries to form as

low-energy dislocation structures (LEDSs) and to maintain their character during further deformation by the formation of ledges (Pantleon & Hansen 1998).

In addition to the general pattern of microstructural evolution, supplementary microstructural features have been observed, for instance as a result of localized shearing both on a macroscale (not confined to a single grain) or on a microscale (Hughes & Hansen 1993; Rosen *et al*. 1995). These different features appear to be common both to single crystals and polycrystals (see, for example, Godfrey *et al*. 1998*a*, *b*; Huang 1998; Huang & Hansen 1997), with the exception of rather narrow zones near grain boundaries and triple junctions, where the evolution in microstructure and local crystallography is affected by grain interaction (Barlow *et al*. 1985; Randle *et al*. 1996). An important aspect of the microstructural evolution is microstructural transitions, for example the transition from a two-dimensional carpet structure (i.e. tilt walls parallel to the primary plane with rotation axis normal to the primary Burgers vector, see Steeds (1966)) in stage II, to a cell-block structure in stage III, and the transition during rolling from a structure of bands (DDWs/MBs) oriented *ca*. 40° to the rolling plane (see figures 1 and 6) to a lamellar structure at large strain oriented almost parallel to the rolling plane (see figure 7). As an illustration, the former transition has been related to the initiation of cross-slip, whereas the latter has been related to the occurrence of localized glide (Hughes & Hansen 1993). Further contributing to the formation of lamellar structures may be the presence of deformation-induced high-angle boundaries that align with the rolling plane at large strain. Future studies may add a new transition that at large strain will transform an oriented structure into an equiaxed subgrain structure. Indication of such a transition is observed in processes where the strain direction is changed (Armstrong *et al*. 1982) or where the deformation processes involve a significant deformation by shear (Harris *et al*. 1996; Valiev *et al*. 1993; Gasperini *et al*. 1996).

Besides the effects of plastic strain, the microstructural subdivision may be significantly affected by the crystallographic orientation of a crystal or a grain. This has been demonstrated in many studies of single crystals (Wrobel *et al*. 1994; Driver *et al*. 1994; Godfrey *et al*. 1998*a*, *b*). For polycrystals, however, a clear relationship between deformation microstructures and grain orientations was not expected, although many studies have stated qualitatively that the microstructure can differ markedly between grains and within grains. It has also been argued that grain–grain interaction may lead to a similar behaviour of all grains, at least at large strains (Beaudoin *et al*. 1996; Dawson & Beaudoin 1998). However, recent experiments in rolled aluminium and aluminium and copper strained in tension (Huang & Hansen 1997; Huang 1998; Liu *et al*. 1998) have shown clear correlations between grain orientation and deformation microstructure. An example of tensile strained copper (grain size *ca*. 100 μm) is given in figure 9. It can be seen that the microstructure is classified into

(i) a cell-block structure with crystallographic DDWs/MBs;

(ii) a cell structure; and

(iii) a cell-block structure with non-crystallographic DDWs/MBs.

A similar relationship to that in figure 9 has been found for polycrystalline aluminium (Huang & Hansen 1997), and for both materials good agreement is found, comparable

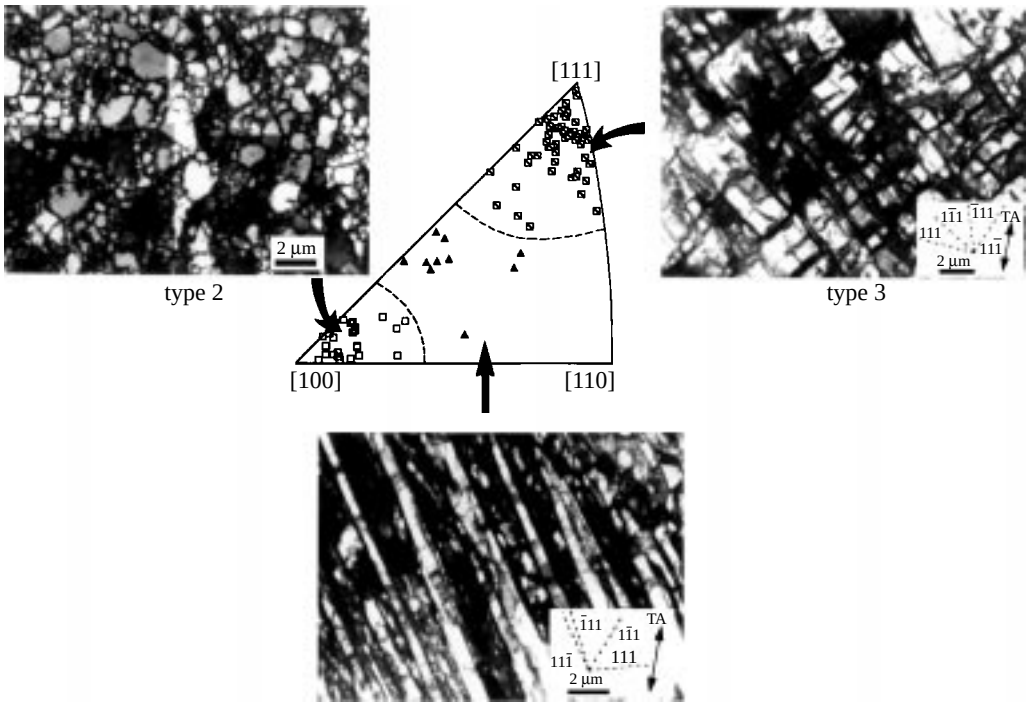

Figure 9. Deformation microstructures for grains of different orientations in polycrystalline specimens (grain size *ca.* 50 μm) of pure copper (99.999%) deformed in tension at room temperature to different strains in the range 0.05–0.28. The tensile axis of the grains is shown in the inverse pole figure, which allows the deformation microstructures (examined by TEM) to be divided into three types. (i) Type 1 (in grains having orientations in the main part of the triangle) is characterized by its content of extended dislocation boundaries following {111} planes. (ii) Type 2 (in grains in the [100] corner) is characterized by its content of cell boundaries. (iii) Type 3 (in grains in the {111} corner) is characterized by its content of extended boundaries that are not on {111} planes (Huang 1998).

with single-crystal experiments. It has also been discovered that structural variations within a single grain can be caused by the subdivision of the grain, whereby different regions of a grain rotate to different orientations.

The orientation effect observed at low and medium strain also appears to affect the structure at large strain. Large variations both in morphology and in local crystallography have been observed (Hughes & Hansen 1993, 1997) that can only be related to an effect of grain orientation. Furthermore, in those structures subdivided by a large fraction of high-angle boundaries (Hughes & Hansen 1997; Juul Jensen *et al*. 1998), it has been shown (Juul Jensen 1997), for 50% and 90% cold rolled aluminium, that there are significant differences between randomly mixed misorientation distributions, calculated from orientations present in the deformation texture, and the experimental distribution for all boundaries determined by Kikuchi line TEM measurement. This is illustrated in figure 10 for 90% cold rolled aluminium. A similar conclusion has been reached by Oscarsson *et al*. (1994). These findings point to structural variations due to an effect of the grain orientation.

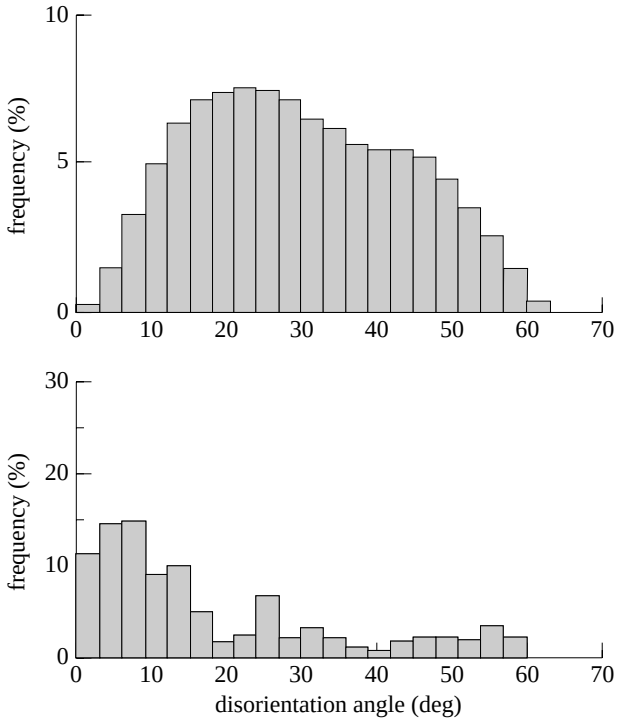

Figure 10. (*a*) Calculated randomly mixed misorientation distribution for commercially pure aluminium cold rolled 90%. (*b*) Experimentally determined misorientation distribution (Juul Jensen 1997).

4. Microstructural parameters

A quantitative description and modelling of the microstructural and crystallographic evolution must be based on a detailed structural characterization, where the dislocation boundaries and the high-angle boundaries are of special interest. These boundaries are characterized by various parameters:

(i) crystallographic orientation (orientation of the boundary plane with respect to a slip plane);

(ii) macroscopic orientation (orientation of the boundary plane with respect to the sample axes);

(iii) spacing;

(iv) morphology;

(v) angle/axis pair (angle at misorientation and misorientation axis between neighbouring crystallites);

(vi) boundary normal;

(vii) boundary character (tilt, twist or mixed); and

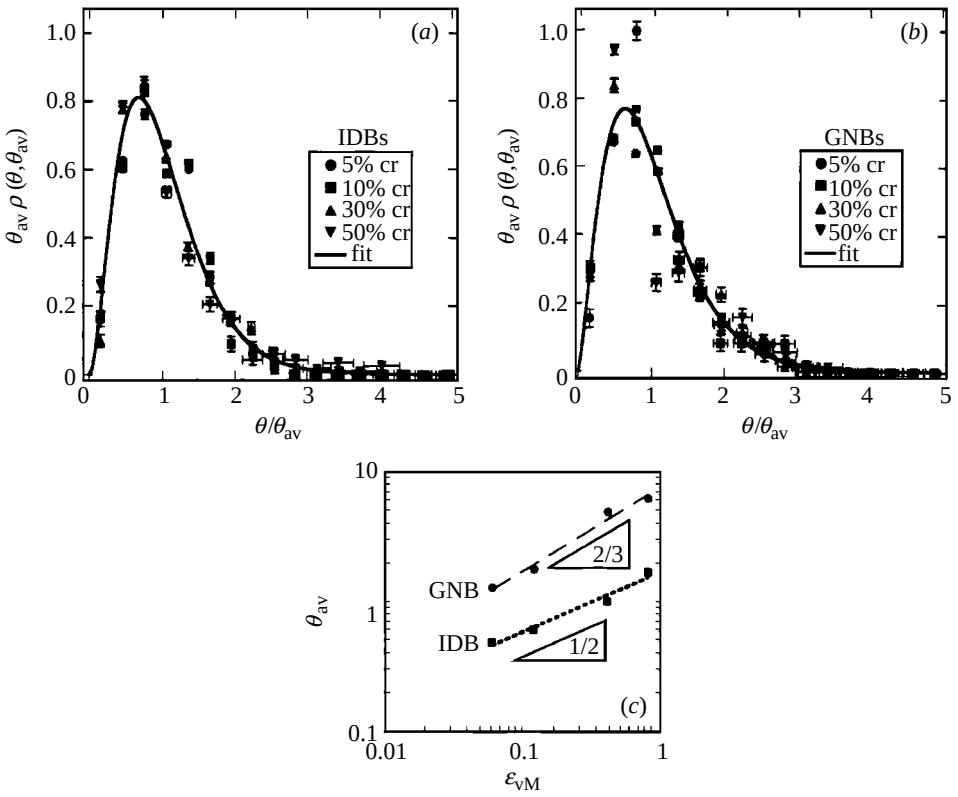

Figure 11. (*a*) and (*b*) show distributions of measured misorientation angles for IDBs and GNB in aluminium following different amounts of rolling reduction, represented by single distributions. (*c*) Average misorientation angles as functions of von Mises strain (ε_{VM}) (Hughes *et al.* 1997).

(viii) dislocation content.

A quantitative determination of the microstructural parameters is time consuming, and application of scaling laws has been suggested (Hughes *et al.* 1997). This method was first applied to the misorientation angle distribution following different amounts of strain. Such a distribution $p(\theta, \theta_{av})\,d\theta$ is defined as the probability that a boundary selected at random has an angle that is between θ and $\theta + d\theta$, given that the average angle is θ_{av}. The distribution depends on the strain, but it was found that, independent of material and deformation mode, the distributions could be represented by a single distribution using the average angle at each strain. It was further found that the scaling hypothesis applied to both GNBs and IDBs when they were analysed separately, but not when all the boundaries were grouped together. An example of the application of this scaling hypothesis to aluminium is shown in figure 11. The successful use of a scaling law has recently been extended to the spacing between the dislocation boundaries (Godfrey & Hughes 1998) and it has been found that the spacing distribution of GNBs in cold rolled aluminium, iron and nickel can be scaled by using the average spacing at each strain.

A further step in applying the scaling hypothesis is to combine distributions of misorientation angles and boundary spacings. These parameters may be related in such

a way that the average misorientation angle is inversely proportional to the boundary spacing. This relationship, i.e. the principle of similitude (Kuhlmann-Wilsdorf 1995), has been found in an analysis of aluminium deformed in tension and rolling at strains from 0.05 to 0.5 (Hansen & Hughes 1995). It was also found that the principle of similitude does not apply for the whole structure, as the cell blocks and the cells evolve differently with increasing strain.

The scaling of structural parameters has been followed by an investigation of the relationship between the misorientation angle and the plastic strain. Experimentally, it has been found for rolled aluminium (Hughes 1995) that the average angle across IDBs and GNBs is proportional to $\varepsilon^{1/2}$ and $\varepsilon^{2/3}$, respectively (see figure 11). Theoretically, for cell boundaries (IDBs), an $\varepsilon^{1/2}$ relationship has been found (Argon & Haasen 1993; Nabarro 1994) assuming a random capture of glide dislocations in the cell walls. A similar result has been obtained by Pantleon (1998), who showed that the $\varepsilon^{1/2}$ relationship only holds for a constant cell size. However, if a dependency of the cell size on the flow stress is taken into account, it has been found that the exponent changes and a value of 0.72 has been found in an analysis of cold rolled aluminium. For GNBs, Basson & Driver (1998) have developed a model for the evolution of GNBs between two crystallites (see § 5 for further details). In this model, the dynamic accumulation of dislocations leads to an increase in the misorientation angle almost proportional to the plastic strain, but with a rate that is very sensitive to the initial grain orientation.

In concluding this section, it must be emphasized that the different types of analysis described above not only have practical relevance but are also of significant theoretical interest, as they can contribute to the understanding of how dislocation boundaries form and develop during straining (see § 2) and how they can affect macroscopic properties, for example strain hardening (see § 6).

5. Microstructure and slip pattern

The discovery of a relationship between the crystal or grain orientation and the evolution of microstructure and crystallography has led to different types of analysis linking the microstructure and the slip pattern.

A basis for this analysis is the assumption that the dislocation structures formed are LEDSs, i.e. the dislocation structures during plastic deformation minimize their free energy per unit length of dislocation line among all configurations accessible to the dislocations. Constraining factors for the formation of these LEDSs can, for example, be the number of available slip systems, dislocation mobility and frictional stress (Kuhlmann-Wilsdorf 1989, 1995, 1999). That the dislocation arrangements are low-energy structures, i.e. free of significant stress–strain fields, is supported by TEM observations showing that there is an abrupt change in electron diffraction contrast on crossing the boundary. It has also been shown, by low-angle X-ray scattering studies (Müller *et al*. 1996; Strauss *et al*. 1996), that long-range internal stresses in deformed metals are relatively small. These indications of the elastic distortion being restricted to the region close to the dislocations are the basis for the analysis of the deformation-induced dislocation boundaries using the Frank formula (Frank 1950).

For a general low-energy boundary with an axis/angle pair (R/θ), this formula (Hull 1975) is

$$\boldsymbol{d} = (\boldsymbol{r} \times \boldsymbol{R}) 2 \sin \tfrac{1}{2}\theta \quad \text{or} \quad \boldsymbol{d} = (\boldsymbol{r} \times \boldsymbol{R})\theta,$$

for small values of θ. The vector $\boldsymbol{r}$ represents an arbitrary straight line lying in the plane of the boundary, which contains the dislocation network, and $\boldsymbol{d}$ is the sum of all the Burgers vectors of all the dislocations intersected by $\boldsymbol{r}$. This equation is the basis of a boundary-slip pattern analysis, of which examples are given below.

In an analysis by Wert *et al*. (1995), the relationship between the slip pattern and the dislocation content in GNBs was examined. This analysis is based on experimental measurements of the axis/angle pair and the boundary normal ($\boldsymbol{n}$). There are two reasons that only GNBs are analysed. GNBs are so straight that n can be determined, and the relatively higher misorientations across GNBs allow an accurate determination of $\boldsymbol{R}$. The boundary parameters are used to calculate the dislocation combinations in the boundary that fit these parameters using the Frank formula. In general, a minimum of three independent Burgers vectors ($\boldsymbol{b}$) are required to form the lattice misorientation of a boundary. For $\boldsymbol{b} = a/2\langle 110\rangle$ in an FCC lattice, 16 independent combinations of three $\boldsymbol{b}$ are possible. To select among these combinations, Wert *et al*. (1995) have suggested that the combinations that give a low energy, i.e. a low density of dislocations, are those to be chosen. The next step is to compare the dislocation content in the boundaries with the $\boldsymbol{b}$-vectors from the active slip systems in the crystallites on each side of the boundary. Based on the orientation, the slip systems are calculated using a full constraint Taylor–Bishop–Hill analysis or a Schmid-factor analysis. Then, for each boundary, the fraction of dislocations in the boundary that originate from the predicted slip systems is calculated, and the combinations that have the highest concentration of $\boldsymbol{b}$ vectors from active slip systems are chosen. This boundary-slip pattern analysis has been applied to a polycrystalline aluminium cold rolled 30% and 50% (Wert *et al*. 1995), and it was found for 17 out of 20 grains that the predominant dislocations in the GNBs coincide with those coming from the slip systems identified by a Schmid-factor analysis. However, it has also been noted that a significant fraction of the dislocations may have their origin in deformation on other systems (unpredicted glide, i.e. not predicted from the applied stress). This approach by Wert *et al*. (1995) has been further extended (Basson & Driver 1998) to consider the dynamic changes in misorientation angles across GNBs at increasing strain. In their analysis, optimal dislocation configurations are obtained by minimizing the dislocation content in the GNB and, at the same time, maximizing the proportion of the boundary dislocations among the slip dislocations derived from a Taylor–Bishop–Hill analysis. This approach has demonstrated a significant effect of the crystal orientation on the evolution of the misorientation angle, in qualitative agreement with experimental observations (Driver *et al*. 1994).

Finally, the effect of grain orientation on the orientation of the GNBs has been analysed in cold rolled aluminium (Liu *et al*. 1998). For one group of grains with similar orientations (type A), the GNBs follow a (111) plane and contain Burgers vectors that belong to the slip plane $(11\bar{1})$. For another group of grains with a different orientation (type B), the GNBs do not follow a slip plane and they have, predominantly, Burgers vectors belonging to two active slip planes. The boundaries have further been analysed by calculating $|\boldsymbol{R}\cdot\boldsymbol{n}|$, which for the former type of boundaries are in the range 0.55–0.93, and for the latter type of boundaries are in the range 0.14–0.33. This means that the crystallographic boundaries have a mixed tilt and twist character, and the non-crystallographic ones are predominantly tilt boundaries. The finding that some grain orientations give rise to the formation of crystallographic GNBs, whereas others do not, has been followed up by a model

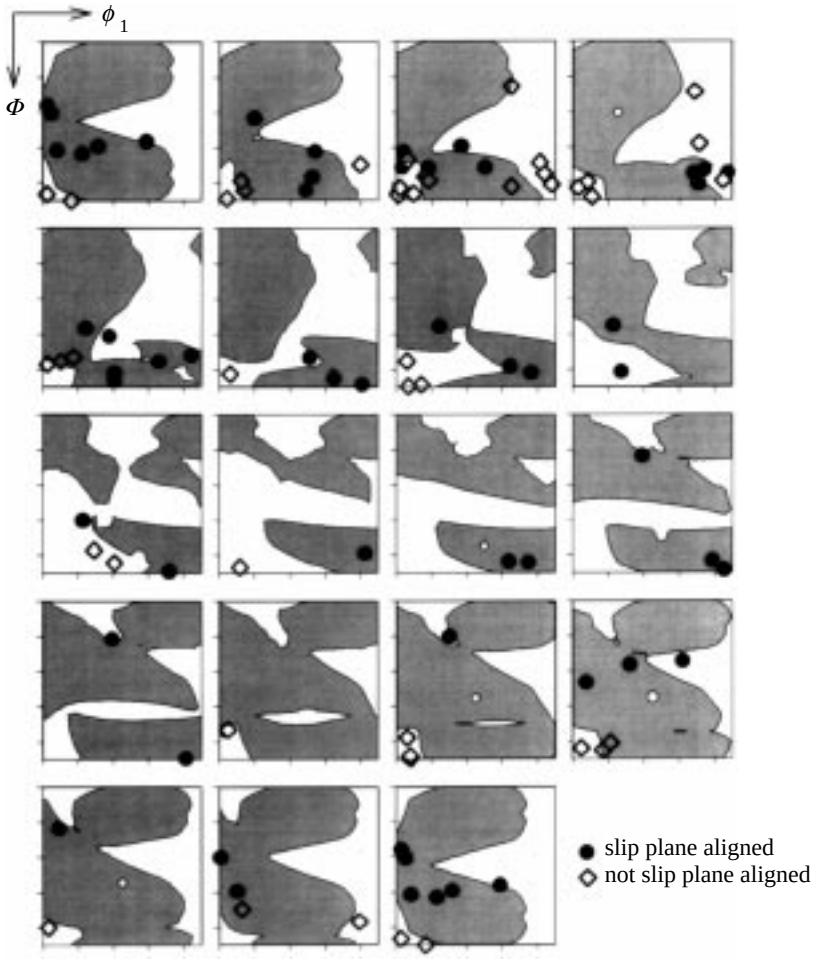

Figure 12. Orientation distribution function (ODF) showing experimental grain orientations with slip-plane-aligned and not-slip-plane-aligned GNBs plotted on top of an ODF, where the grey areas are associated with high slip activity in rolled FCC materials on two sets of coplanar slip systems in the slip planes with which the boundaries are aligned (courtesy of G. Winther).

to predict the orientations of grain in which the GNBs will follow the (111) planes (Winther *et al*. 1997). Using this model, it was predicted that crystallographic GNBs form if two active slip systems in the same slip plane account for a large fraction of the slips in the grain. This finding is illustrated in figure 12. Improvement of this model is currently in progress in order to include deformation in tension and to model misorientations across dislocation boundaries (Winther 1998).

In concluding this section, the correlations between

(i) microstructure/grain orientation;

(ii) grain orientation/slip pattern; and

(iii) slip pattern/microstructure,

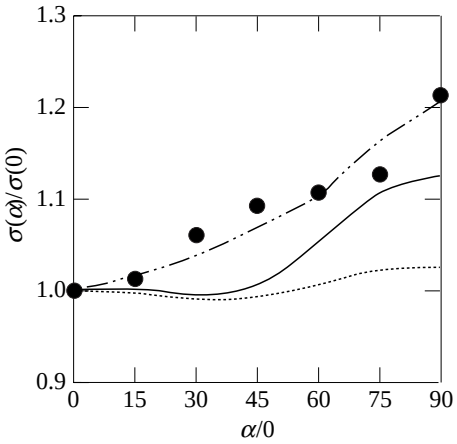

Figure 13. Samples for uniaxial tensile testing cut out of a rolled sheet with different angles, α, between the rolling and tensile directions. Comparison of normalized yield stress data for aluminium with prediction of the full constraint Taylor model and two models assuming that dislocation boundaries (GNBs) both macroscopically oriented and slip-plane aligned introduce flow stress anisotropy (Winther 1998). Filled circles, experimental results; dotted line, full constraint Taylor model; solid line, texture plus dislocation structure (five-slip systems); dashed–dotted line, texture plus dislocation structure (one-slip system).

have led to a consideration of a coupling of the microstructural and texture evolution (Winther *et al*. 1997; Winther 1998). Consequently, a change in grain orientation may lead to a change in microstructure. The microstructure formed will affect the slip pattern, which, in turn, will affect the lattice rotation. A modelling of these coupled evolutions will require an updating of the microstructural evolution with increasing strain, in the same way as texture models update lattice rotations. Elements of such an update are described above and in § 4.

6. Macroscopic properties

The suggested slip pattern leading to grain subdivision on different length-scales by dislocation boundaries of different characteristics will have important effects on the macroscopic properties.

The flow stress may be somewhat reduced due to a reduction in the number of slip systems compared to homogeneous deformation. The arrangement of dislocations into IDBs and GNBs might be of great importance, and may be characterized by a misorientation angle that increases with an increase in stress and strain (Liu & Hansen 1995). Thereby, the strength of these boundaries increases, which will affect both flow stress (Hansen & Huang 1998) and flow stress anisotropy (see figure 13). The observed structural transitions at increasing strain, which suggest correlations between such transitions, and the observation of work hardening stages are also important.

The deformation texture may also be affected by grain subdivision allowing a relaxation of some constraints (Lee & Duggan 1993; Leffers 1993; Kocks & Canova 1981). This will lead to a modified texture evolution, for instance, when compared to that predicted by a Taylor–Bishop–Hill model assuming homogeneous deforma-

tion of the individual grains. The grain subdivision will also cause a slower texture evolution and a larger texture spread than that obtained in conventional texture models. Further, the observed correlation between microstructure and slip pattern may lead to a coupling of the texture and microstructure evolution. By taking such a coupling into account, some discrepancies may be removed between present-day texture models and texture measurements.

Finally, the microstructural evolution leading to a break-up on a fine scale by dislocation boundaries and high-angle boundaries will affect both the number and distribution of potential nucleation sites, and the magnitude and distribution of the stored energy. For experiments and modelling related to this, see Juul Jensen (1995*a*, *b*).

7. Conclusions

The analysis of the orientation dependencies of the structural evolution by grain subdivision on a finer and finer scale has demonstrated that crystal plasticity models can be used to combine structural observations with plastic deformation behaviour. It has also been found that structural information can be analysed by applying scaling laws. Integration of these approaches and their extension to include atomistic modelling and finite-element modelling will, therefore, lead us towards an understanding of plastic deformation and the significant property changes that form part of this deformation.

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